

CONSTRAINTS ON FERMION MAGNETIC AND ELECTRIC MOMENTS FROM LEP-I*

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ABSTRACT

The effective Lagrangian approach allows us to constrain fermion magnetic and electric moments using LEP-I data. We improve some of the previous limits on these moments.

[†] Research supported by a F.P.I. grant from the Universitat Autònoma de Barcelona.

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* Work partially supported by the research project CICYT-AEN90-0019.

1 Introduction

The magnetic and electric dipole moments of fermions contain important information about their nature and interactions. On the one hand, concerning the magnetic moments, the precise measurements of the $g - 2$ values of the electron and the muon provide, among other things, an accurate test of the point-like character of both leptons. However, limits on the magnetic dipole moment of the tau lepton, of the neutrinos and of the different quark species are much poorer.

On the other hand, the most stringent limits on electric dipole moments concern the neutron and the electron, and this sets strong constraints on some of the CP-violating parameters. Using quark-model arguments, one is able to infer upper bounds on the electric dipole moments of the up and down quarks from the neutron measurements. However, again the electric dipole moments of the tau lepton, of the neutrinos, and of the second and third generation quarks are much less constrained.

The aim of the present paper is to extract “indirect” limits on fermion moments using the electroweak data, in the context of an effective Lagrangian approach. Our main idea is fairly simple. The effective Lagrangian that may induce fermion moments different from the Standard Model expectations inevitably induces anomalous couplings of the neutral boson Z to fermions. We will obtain constraints on these anomalous couplings using the available electroweak data, and these constraints will provide bounds on the fermion magnetic and electric moments.

Let us define here the general electromagnetic matrix element describing the interaction of a fermion with the photon. For a charged fermion one usually defines

$$\begin{aligned} \langle p_2 | J_{em}^\mu(0) | p_1 \rangle &= -e Q_f \bar{u}(p_2) \left(F_1^f \gamma^\mu + \frac{i}{2m_f} F_2^f \sigma^{\mu\nu} q_\nu \right) u(p_1) \\ &+ e \bar{u}(p_2) F_3^f \gamma_5 \sigma^{\mu\nu} q_\nu u(p_1) , \end{aligned} \tag{1}$$

where $e Q_f$ is the fermion charge, and $q = p_2 - p_1$. The form factors F_2 and F_3 , evaluated in

the static limit, correspond to the anomalous magnetic moment

$$a_f = F_2^f(q^2 = 0) , \quad (2)$$

and to the electric dipole moment

$$d_f = e F_3^f(q^2 = 0) . \quad (3)$$

The electromagnetic interactions of neutrinos are described in terms of

$$\langle p_2 | J_{em}^\mu(0) | p_1 \rangle = \bar{u}(p_2) (i F_2^\nu \mu_B + e F_3^\nu \gamma_5) \sigma^{\mu\nu} q_\nu u(p_1) , \quad (4)$$

with $\mu_B = \frac{e}{2m_e}$ the Bohr magneton. In units of μ_B , the neutrino magnetic moment is

$$\kappa_\nu = F_2^\nu(q^2 = 0) , \quad (5)$$

and the electric dipole moment is

$$d_\nu = e F_3^\nu(q^2 = 0) . \quad (6)$$

In the following section we discuss the general analysis of the fermion moments in the linear effective Lagrangian approach, and in section 3 we describe our procedure to extract our limits and find them. The non-linear effective Lagrangian approach is discussed in section 4. In section 5 we present our main conclusions.

2 Linear effective Lagrangian analysis of fermion moments

Deviations from the electroweak Standard Model (SM) realized in its minimal linear form, with a perturbative scalar sector, can be treated by using effective Lagrangians. The general idea of the linear effective Lagrangian approach is that theories beyond the SM, emerging at some characteristic energy scale Λ , have effects at low energies $E \leq G_F^{-1/2}$, and these effects can be taken into account by considering a Lagrangian that extends the SM Lagrangian, \mathcal{L}_{SM} :

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{\lceil\{\{ . \quad (7)$$

The effective Lagrangian $\mathcal{L}_{\lceil\{\{$ contains operators of increasing dimension that are built with the SM fields including the scalar sector, and is organized as an expansion in powers of $(1/\Lambda)$.

The success of the SM at the level of quantum corrections can be considered as a check of the gauge symmetry properties of the model. To preserve the consistency of the low energy theory, with a Lagrangian given by Eq. (7), we will assume that \mathcal{L}_{eff} is $SU(3) \otimes SU(2) \otimes U(1)$ gauge invariant. Some of the problems that originate when dealing with non-gauge invariant interactions have been discussed in [1, 2]. The gauge-invariant operators that dominate at low energies have dimension 6 and have been listed in [3]. We will now write the set of operators that would contribute to the dipole moments we are interested in. For each one of the listed operators, its hermitian conjugate will also contribute to the corresponding moment. We will not display the hermitian conjugate, although we considered it in our calculations.

Given a charged lepton ℓ , there are two operators contributing to its magnetic dipole moment

$$\begin{aligned}\mathcal{O}_{\ell\mathcal{B}} &= \bar{L} \sigma^{\mu\nu} \ell_R \Phi B_{\mu\nu} , \\ \mathcal{O}_{\ell\mathcal{W}} &= \bar{L} \sigma^{\mu\nu} \vec{\sigma} \ell_R \Phi \vec{W}_{\mu\nu} ,\end{aligned}\tag{8}$$

where L is the lepton isodoublet containing ℓ and ℓ_R the singlet partner. As we will see, the operators will involve the $U(1)$ and $SU(2)$ field strengths,

$$\begin{aligned}B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu , \\ W_{\mu\nu}^i &= \partial_\mu W_\nu^i - \partial_\nu W_\mu^i - g\epsilon_{jk}^i W^j W^k ,\end{aligned}\tag{9}$$

as well as the Higgs field Φ or its conjugate $\tilde{\Phi} = i\sigma_2\Phi^*$. When considering quarks we have to distinguish between up-type and down-type induced magnetic moments. When the quark is $U = u, c$ or t , we have contributions coming from

$$\begin{aligned}\mathcal{O}_{U\mathcal{B}} &= \bar{Q} \sigma^{\mu\nu} U_R \tilde{\Phi} B_{\mu\nu} , \\ \mathcal{O}_{U\mathcal{W}} &= \bar{Q} \sigma^{\mu\nu} \vec{\sigma} U_R \tilde{\Phi} \vec{W}_{\mu\nu} ,\end{aligned}\tag{10}$$

where Q is the corresponding quark isodoublet, while for the case $D = d, s$ or b one has

$$\begin{aligned}\mathcal{O}_{D\mathcal{B}} &= \bar{Q} \sigma^{\mu\nu} D_R \Phi B_{\mu\nu} , \\ \mathcal{O}_{D\mathcal{W}} &= \bar{Q} \sigma^{\mu\nu} \vec{\sigma} D_R \Phi \vec{W}_{\mu\nu} .\end{aligned}\tag{11}$$

Let us now display the operators contributing to the electric dipole moments. For the charged leptons we have

$$\begin{aligned}\tilde{\mathcal{O}}_{\ell\mathcal{B}} &= \bar{L} \sigma^{\mu\nu} i \gamma_5 \ell_R \Phi B_{\mu\nu} , \\ \tilde{\mathcal{O}}_{\ell\mathcal{W}} &= \bar{L} \sigma^{\mu\nu} i \gamma_5 \vec{\sigma} \ell_R \Phi \vec{W}_{\mu\nu} ,\end{aligned}\tag{12}$$

while for $U = u, c, t$ we have

$$\begin{aligned}\tilde{\mathcal{O}}_{UB} &= \bar{Q} \sigma^{\mu\nu} i \gamma_5 U_R \tilde{\Phi} B_{\mu\nu} , \\ \tilde{\mathcal{O}}_{UW} &= \bar{Q} \sigma^{\mu\nu} i \gamma_5 \vec{\sigma} U_R \tilde{\Phi} \vec{W}_{\mu\nu} ,\end{aligned}\tag{13}$$

and finally, for $D = d, s, b$

$$\begin{aligned}\tilde{\mathcal{O}}_{DB} &= \bar{Q} \sigma^{\mu\nu} i \gamma_5 D_R \Phi B_{\mu\nu} , \\ \tilde{\mathcal{O}}_{DW} &= \bar{Q} \sigma^{\mu\nu} i \gamma_5 \vec{\sigma} D_R \Phi \vec{W}_{\mu\nu} .\end{aligned}\tag{14}$$

The case of the neutrino dipole moments has to be treated separately. We need to enlarge the minimal SM by adding the right-handed neutrinos $\nu_{eR}, \nu_{\mu R}, \nu_{\tau R}$. Once this is done, we have the following operators contributing to the neutrino magnetic moment

$$\begin{aligned}\mathcal{O}_{\nu_\ell \mathcal{B}} &= \bar{L} \sigma^{\mu\nu} \nu_{\ell R} \tilde{\Phi} B_{\mu\nu} , \\ \mathcal{O}_{\nu_\ell \mathcal{W}} &= \bar{L} \sigma^{\mu\nu} \vec{\sigma} \nu_{\ell R} \tilde{\Phi} \vec{W}_{\mu\nu} ,\end{aligned}\tag{15}$$

and to the neutrino electric dipole moment

$$\begin{aligned}\tilde{\mathcal{O}}_{\nu_\ell B} &= \bar{L} \sigma^{\mu\nu} i \gamma_5 \nu_{\ell R} \tilde{\Phi} B_{\mu\nu} , \\ \tilde{\mathcal{O}}_{\nu_\ell W} &= \bar{L} \sigma^{\mu\nu} i \gamma_5 \vec{\sigma} \nu_{\ell R} \tilde{\Phi} \vec{W}_{\mu\nu} .\end{aligned}\tag{16}$$

In (15) and (16), L is the lepton isodoublet containing ν_ℓ .

The effective Lagrangian to be considered can now be written as a linear combination of the operators we have listed

$$\begin{aligned}\mathcal{L}_{\mathcal{H}\mathcal{H}} &= \sum_f \left(\frac{\alpha_{fB}}{\Lambda^2} \mathcal{O}_{\mathcal{B}} + \frac{\alpha_{fW}}{*\in} \mathcal{O}_{\mathcal{W}} \right) \\ &+ \sum_f \left(\frac{\tilde{\alpha}_{fB}}{\Lambda^2} \tilde{\mathcal{O}}_{fB} + \frac{\tilde{\alpha}_{fW}}{\Lambda^2} \tilde{\mathcal{O}}_{fW} \right) .\end{aligned}\tag{17}$$

It is clear that below the scale of spontaneous symmetry breaking the effective Lagrangian in Eq.(17) induces contributions to the anomalous magnetic moments a_f and the electric moments d_f . Substituting

$$\Phi \longrightarrow \begin{pmatrix} 0 \\ v \\ \frac{v}{\sqrt{2}} \end{pmatrix} ,\tag{18}$$

with $v^2 = 1/(\sqrt{2}G_F) \simeq (246 \text{ GeV})^2$, one gets the following contributions

$$\delta a_f = -2\sqrt{2} \frac{m_f}{v} \frac{1}{e Q_f} (c_w \epsilon_{fB} + s_w \epsilon_{fW}) ,\tag{19}$$

for any charged fermion f being the up-type component of the lepton and quark isodoublets, and

$$\delta a_f = -2\sqrt{2} \frac{m_f}{v} \frac{1}{eQ_f} (c_w \epsilon_{fB} - s_w \epsilon_{fW}) , \quad (20)$$

for the down-type components. We have defined the dimensionless parameters

$$\begin{aligned} \epsilon_{fB} &= \alpha_{fB} \frac{v^2}{\Lambda^2} , \\ \epsilon_{fW} &= \alpha_{fW} \frac{v^2}{\Lambda^2} , \end{aligned} \quad (21)$$

valid for any fermion, and $c_w = \cos\theta_w$, $s_w = \sin\theta_w$. When $f = \nu$ we obtain the following contribution to the neutrino magnetic moment

$$\delta\kappa_\nu = 2\sqrt{2} \frac{m_e}{v} \frac{1}{e} (c_w \epsilon_{\nu B} + s_w \epsilon_{\nu W}) . \quad (22)$$

Finally, for the up-type fermions f one gets the following contribution to d_f

$$\delta d_f = -\frac{\sqrt{2}}{v} (c_w \tilde{\epsilon}_{fB} + s_w \tilde{\epsilon}_{fW}) , \quad (23)$$

and for the down-type fermions

$$\delta d_f = -\frac{\sqrt{2}}{v} (c_w \tilde{\epsilon}_{fB} - s_w \tilde{\epsilon}_{fW}) , \quad (24)$$

where

$$\begin{aligned} \tilde{\epsilon}_{fB} &= \tilde{\alpha}_{fB} \frac{v^2}{\Lambda^2} , \\ \tilde{\epsilon}_{fW} &= \tilde{\alpha}_{fW} \frac{v^2}{\Lambda^2} . \end{aligned} \quad (25)$$

We have discussed in this section how fermion moments are described by linear effective Lagrangians. We will now address our attention to describe the procedure to extract limits in this case, and leave the discussion of non-linear effective Lagrangians until section 4.

3 Bounds on the magnetic and electric moments

We have seen in the last section that the linear effective Lagrangian in Eq. (17) induces fermion moments to be added to the SM contributions. However, the new operators in the effective Lagrangian have other effects. The most interesting, from the phenomenological point of

view, is the anomalous coupling of the Z boson to fermions

$$\begin{aligned}
\mathcal{L} = & \sum_{f=up-type} \frac{\sqrt{2}}{v} (s_w \epsilon_{fB} - c_w \epsilon_{fW}) \bar{f} \sigma^{\mu\nu} f \partial_\nu Z_\mu \\
& + \sum_{f=up-type} \frac{\sqrt{2}}{v} (s_w \tilde{\epsilon}_{fB} - c_w \tilde{\epsilon}_{fW}) \bar{f} \sigma^{\mu\nu} i \gamma_5 f \partial_\nu Z_\mu \\
& + \sum_{f=down-type} \frac{\sqrt{2}}{v} (s_w \epsilon_{fB} + c_w \epsilon_{fW}) \bar{f} \sigma^{\mu\nu} f \partial_\nu Z_\mu \\
& + \sum_{f=down-type} \frac{\sqrt{2}}{v} (s_w \tilde{\epsilon}_{fB} + c_w \tilde{\epsilon}_{fW}) \bar{f} \sigma^{\mu\nu} i \gamma_5 f \partial_\nu Z_\mu + \dots .
\end{aligned} \tag{26}$$

These anomalous couplings would shift the partial widths $\Gamma(Z \rightarrow f\bar{f}) \equiv \Gamma_f$ from their predicted values in the Standard Model

$$\Gamma_f = \Gamma_f^{SM} + \delta\Gamma_f . \tag{27}$$

The agreement between experiment and the SM predictions implies limitations on the strength of the different *a priori* independent terms in the effective Lagrangian. We will not allow for unnatural cancellations of the effects produced by different operators, and thus we will consider one operator at a time.

We should emphasize that once we have selected one of the contributions to a particular fermionic width Γ_f , it is not enough to simply use the experimental result Γ_f^{exp} to get our constraints. This is due to the well-known m_t -dependence of the theoretical predictions of the SM, and to a less extent to the dependence on the Higgs mass M_H . We choose to keep the m_t -dependence of our results explicitly and allow M_H to span the range 60 GeV to 1 TeV, with this “theoretical uncertainty” (and the theoretical uncertainty in the experimental value of α_s) linearly summed to the experimental errors. It is quite common to combine theoretical and experimental errors in quadrature. Here, we adopt the more conservative point of view of adding the two types of uncertainties *linearly*.

For the theoretical SM predictions we borrow the results of Bardin *et al.* [4], that were kindly made available to us by M.Bilenky.

We use the LEP value $M_Z = 91.187 \text{ GeV}$ as input, as well as the current values of α and G_F [5, 6]. The observables we use to constrain deviations from the Standard Model are:

1) the experimental LEP-I data [5]

$$\begin{aligned}
\Gamma_e &= 83.86 \pm 0.30 \text{ MeV} , \\
\Gamma_\mu &= 83.78 \pm 0.40 \text{ MeV} , \\
\Gamma_\tau &= 83.50 \pm 0.45 \text{ MeV} , \\
\Gamma_{inv} &= 497.6 \pm 4.3 \text{ MeV} , \\
\Gamma_{had} &= 1740.3 \pm 5.9 \text{ MeV} , \\
g_{A_e} &= -0.50096 \pm 0.00093 , \\
g_{A_\mu} &= -0.5013 \pm 0.0012 , \\
g_{A_\tau} &= -0.5005 \pm 0.0014 ,
\end{aligned} \tag{28}$$

2) the W -mass determined from the ratio M_W/M_Z measured at $p\bar{p}$ colliders and the LEP value for M_Z [7]

$$M_W = 80.24 \pm 0.09^{+0.01}_{-0.02} \text{ GeV} , \tag{29}$$

3) the ratio of inclusive neutral- to charged-currents neutrino cross sections on approximately isoscalar targets [8]

$$R_\nu = \frac{\sigma(\nu N \rightarrow \nu + \dots)}{\sigma(\nu N \rightarrow \mu + \dots)} = 0.308 \pm 0.002 , \tag{30}$$

and

4) the CDF limit on the top quark mass [9]

$$m_t \geq 108 \text{ GeV} . \tag{31}$$

We now turn our attention to the explicit expressions for $\delta\Gamma_f$ due to the novel effects we are considering. Starting with the operators $\mathcal{O}_{\{B\}}$, we have that for each fermion f the relative width shift is

$$\frac{\delta\Gamma_f}{\Gamma_f} = \frac{s_w^2}{v_f^2 + a_f^2} \epsilon_{fB}^2 , \tag{32}$$

where $v_f = T_3^f - 2s_w^2 Q_f$ and $a_f = T_3^f$. The operators $\mathcal{O}_{\{W\}}$ induce the change

$$\frac{\delta\Gamma_f}{\Gamma_f} = \frac{c_w^2}{v_f^2 + a_f^2} \epsilon_{fW}^2 . \tag{33}$$

In the CP-violating sector, the operators $\tilde{\mathcal{O}}_{fB}$ lead to

$$\frac{\delta\Gamma_f}{\Gamma_f} = \frac{s_w^2}{v_f^2 + a_f^2} \tilde{\epsilon}_{fB}^2 , \tag{34}$$

while the type \tilde{O}_{fW} lead to

$$\frac{\delta\Gamma_f}{\Gamma_f} = \frac{c_w^2}{v_f^2 + a_f^2} \tilde{c}_{fW}^2 . \quad (35)$$

As we have already discussed, the limits on the different parameters ϵ in Eqs. (32,33) and $\tilde{\epsilon}$ in Eqs. (34,35) will depend sensitively on m_t , since we have chosen to combine the weak dependence on M_H linearly with the experimental uncertainties. The limits are obtained by comparing the experimental values of the observables in Eqs. (28–31) with the predictions that we get when adding the SM results and the corrections previously discussed. Our results on ϵ_{fB} are presented in Fig. 1, and the ones on ϵ_{fW} in Fig. 2. The projection on the ϵ^2 (or the m_t^2) axis corresponds to a single-variable 68%, or 1σ , confidence-level interval. These limits can be read from Figs. 1 and 2

$$\begin{aligned} \epsilon_{eB}^2 &\leq 1.10 \times 10^{-2} & \epsilon_{eW}^2 &\leq 2.98 \times 10^{-3} , \\ \epsilon_{\mu B}^2 &\leq 1.13 \times 10^{-2} & \epsilon_{\mu W}^2 &\leq 3.05 \times 10^{-3} , \\ \epsilon_{\tau B}^2 &\leq 1.07 \times 10^{-2} & \epsilon_{\tau W}^2 &\leq 2.89 \times 10^{-3} , \\ \epsilon_{UB}^2 &\leq 7.17 \times 10^{-2} & \epsilon_{UW}^2 &\leq 1.93 \times 10^{-2} , \\ \epsilon_{DB}^2 &\leq 7.17 \times 10^{-2} & \epsilon_{DW}^2 &\leq 1.93 \times 10^{-2} , \\ \epsilon_{\nu B}^2 &\leq 4.31 \times 10^{-2} & \epsilon_{\nu W}^2 &\leq 1.16 \times 10^{-2} . \end{aligned} \quad (36)$$

As expected from a comparison of Eqs. (32) and (33) the limits coming from the operators O_{fW} are stronger than the ones coming from O_{fB} . Introducing the limits on ϵ into Eqs. (19,20) we get limits on the anomalous magnetic moments. Looking at this equation, we anticipate that the operators O_{fW} will lead to the tightest bounds on δa_f . We only quote here these strongest limits (we use $m_u = 5 \text{ MeV}$, $m_d = 10 \text{ MeV}$, $m_c = 1.5 \text{ GeV}$, $m_s = 200 \text{ MeV}$, $m_b = 5.0 \text{ GeV}$)

$$\begin{aligned} \delta a_\tau &\leq 6.2 \times 10^{-3} , \\ \delta a_u &\leq 2.3 \times 10^{-5} , \\ \delta a_d &\leq 9.0 \times 10^{-5} , \\ \delta a_c &\leq 6.8 \times 10^{-3} , \\ \delta a_s &\leq 1.8 \times 10^{-3} , \\ \delta a_b &\leq 4.5 \times 10^{-2} , \\ \delta \kappa_{\nu_\tau} &\leq 3.6 \times 10^{-6} . \end{aligned} \quad (37)$$

We have skipped the limits on $\delta a_e, \delta a_\mu$ since the direct measurements make our limits completely useless. The constraints on κ_ν for ν_e and ν_μ coming from Red Giant observations [20] also make our corresponding limits far away from being competitive.

We follow an identical procedure to constrain $\tilde{\epsilon}_{fB}$ and $\tilde{\epsilon}_{fW}$. It turns out that the limits on $\tilde{\epsilon}_{fB}$ are the same than the ones on ϵ_{fB} , and similarly the $\tilde{\epsilon}_{fW}$ and ϵ_{fW} are identical. Again we only quote our best limits on the electric dipole moments (in $e\text{-cm}$)

$$\begin{aligned}
\delta d_\tau &\leq 3.4 \times 10^{-17} , \\
\delta d_c &\leq 8.9 \times 10^{-17} , \\
\delta d_s &\leq 8.9 \times 10^{-17} , \\
\delta d_b &\leq 8.9 \times 10^{-17} , \\
\delta d_{\nu_\tau} &\leq 6.9 \times 10^{-17} .
\end{aligned} \tag{38}$$

Here we have skipped the limits on $\delta d_e, \delta d_\mu, \delta d_u$ and δd_d since the experimental bounds on them are much stronger. Again Red Giant evolution analysis imply much tighter limits for ν_e and ν_μ than our corresponding results.

We finally turn our attention to a comparison of our limits with other results in the literature. We present the comparison in Table 1 for the anomalous magnetic moment and in Table 2 for the electric dipole moment. We improve previous limits for $a_\tau, a_u, a_d, a_c, a_s, a_b, d_\tau, d_{\nu_\tau}$. Bounds on the tau-lepton moments following the approach presented in this section were calculated in Ref. [21].

4 Non-linear effective Lagrangian approach

In section 2 we discussed the case where we extend a linearly realized Standard Model. However, there are models –as the technicolor ones– that do not fit into the linear framework. The assumptions of avoiding the presence of the physical scalar sector, or of sending its mass to infinity do not commute with the linear expansion since Φ is no longer there to construct gauge-invariant effective operators. In this case, $\Lambda \sim 4\pi v$, and it is appropriate to use a non-linear or chiral realization of the Standard Model, and to extend it using non-linear effective Lagrangians. These were discussed in detail back in 1980 in [22] and have been recently reviewed in [23].

We would like to investigate the consequences of adopting the chiral approach for the fermion moments. Let us start with the magnetic moment of the up and down quarks. By working this specific example we will be able to understand the general pattern.

We add a piece to the SM Lagrangian that contains vertices corresponding to a_u and a_d .

The leading terms in the chiral expansion are given by

$$\begin{aligned}\mathcal{L}_{\lceil\{\{} &= \frac{g' \beta'}{v^2} \bar{\psi}_R M \hat{B}^{\mu\nu} \sigma_{\mu\nu} \Sigma^\dagger \psi_L \\ &+ \frac{g \beta}{v^2} \bar{\psi}_R M \Sigma^\dagger \hat{W}^{\mu\nu} \sigma_{\mu\nu} \psi_L + h.c. ,\end{aligned}\tag{39}$$

where

$$\begin{aligned}\psi_{L,R} &= \begin{pmatrix} u \\ d \end{pmatrix}_{L,R} , \\ \hat{W}_{\mu\nu} &= \vec{W}_{\mu\nu} \cdot \vec{\sigma} , \\ \hat{B}_{\mu\nu} &= B_{\mu\nu} \cdot \sigma_3 ,\end{aligned}\tag{40}$$

M is the mass matrix

$$\begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} ,\tag{41}$$

and Σ contains the would-be Goldstone bosons

$$\Sigma = \exp \left\{ i \frac{\vec{\xi} \cdot \vec{\sigma}}{v} \right\} .\tag{42}$$

The contributions to the magnetic dipole moment coming from $\mathcal{L}_{\lceil\{\{}$ will contain the coefficients β and β' in Eq. (39). In the unitary gauge, these contributions can be put in the form of those we found in the linear case, Eqs. (19,20,22,23,24), provided we identify

$$\begin{aligned}\epsilon_{uB} &\rightarrow \frac{\sqrt{2}}{v} g' \beta' m_u , \\ \epsilon_{uW} &\rightarrow \frac{\sqrt{2}}{v} g \beta m_u , \\ \epsilon_{dB} &\rightarrow -\frac{\sqrt{2}}{v} g' \beta' m_d , \\ \epsilon_{dW} &\rightarrow \frac{\sqrt{2}}{v} g \beta m_d .\end{aligned}\tag{43}$$

For our phenomenological purposes, it is interesting to notice that with the very same identifications made in the last equation, our Eq. (26) containing the anomalous couplings to the Z -boson is valid. We can use now the fact, shown in the last section, that our constraints on the anomalous magnetic moment from LEP-1 data do not depend on the definitions of the parameters ϵ . This is also true in the case of the dipole electric moments, where we should use the effective Lagrangian

$$\begin{aligned}\mathcal{L}_{\lceil\{\{} &= \frac{g' \tilde{\beta}'}{v^2} \bar{\psi}_R M \hat{B}^{\mu\nu} \sigma_{\mu\nu} i \gamma_5 \Sigma^\dagger \psi_L \\ &+ \frac{g \tilde{\beta}}{v^2} \bar{\psi}_R M \Sigma^\dagger \hat{W}^{\mu\nu} \sigma_{\mu\nu} i \gamma_5 \psi_L + h.c. .\end{aligned}\tag{44}$$

Here we have worked out the example of the magnetic moments of the u and d quarks. It is clear that it can be easily extended to the magnetic and electric moments of all fermions. Notice also that our final limits do not depend on the mass appearing in the mass matrix in Eq. (41).

Our conclusion is that our limits on fermion magnetic and electric moments derived in a linear effective Lagrangian approach hold true when using the chiral expansion of the non-linear effective Lagrangian.

5 Conclusions

Physics beyond the Standard Model of the electroweak interactions can be described in a general way using effective Lagrangians. We use this approach to study magnetic and electric moments of the fermions. We find that any contribution to these moments inevitably induces anomalous couplings of the Z -boson to fermions.

The LEP-I data severely constrains these anomalous couplings, and the constraints in turn set stringent bounds on the magnetic and electric moments. Some of the bounds represent an improvement on previously derived limits on fermion moments.

We have studied both the linear and non-linear realizations of effective Lagrangians and have shown that both lead to the same numerical limits on moments.

Acknowledgments

One of us (R.E.) would like to thank M. Martínez and F. Teubert for their comments and helpful collaboration in providing the most recent LEP-I data. A special thank goes to C. Ayala and T. Coarasa for \LaTeX macros used in this work. I also acknowledge financial support by a F.P.I. grant from the Universitat Autònoma de Barcelona.

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Table Captions

Table 1: Values of the anomalous magnetic moments excluded by our analysis using LEP-I data, compared to other limits we have found in the literature.

Table 2: Same than Table 1 for the electric moments.

Figure Captions

Figure 1: Allowed regions in the (m_t^2, ϵ_{fB}^2) plane for $f = e$ (**a**), $f = \mu$ (**b**), $f = \tau$ (**c**), $f = U$ (**d**), $f = D$ (**e**), $f = \nu$ (**f**). Projection on the ϵ^2 axis corresponds to a 1σ CL interval.

Figure 2: Same than Figure 1 for the parameters ϵ_{fW}^2 .

a_f	OUR LIMITS	OTHER LIMITS
τ	0.0062	0.02 [10] 0.11 [11] 0.39 ± 0.30 [12]
ν_τ	$3.6 \cdot 10^{-6}$	$5.4 \cdot 10^{-7}$ [13] $4 \cdot 10^{-6}$ [14] $8 \cdot 10^{-6}$ [15]
u	$2.3 \cdot 10^{-5}$	$3 \cdot 10^{-4}$ [10]
d	$9.0 \cdot 10^{-5}$	$6 \cdot 10^{-4}$ [10]
c	$6.8 \cdot 10^{-3}$	0.030 [10]
s	$1.8 \cdot 10^{-3}$	0.025 [10]
b	$4.5 \cdot 10^{-2}$	0.13 [10] -0.34 ± 0.42 [12]

Table 1:

d_f	OUR LIMITS (units $e\text{-}cm$)	OTHER LIMITS (units $e\text{-}cm$)
τ	$3.4 \cdot 10^{-17}$	$1.2 \cdot 10^{-16}$ [16] $1.4 \cdot 10^{-16}$ [17, 18] $6 \cdot 10^{-16}$ [11]
ν_τ	$6.9 \cdot 10^{-17}$	$1.6 \cdot 10^{-16}$ [15]
c	$8.9 \cdot 10^{-17}$	$6.5 \cdot 10^{-23}$ [19]
s	$8.9 \cdot 10^{-17}$	$1.0 \cdot 10^{-24}$ [19]
b	$8.9 \cdot 10^{-17}$	$7.8 \cdot 10^{-21}$ [19]

Table 2:

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